



# Cylindrical milling tools: Comparative real case study for process stability

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## ARTICLE INFO

### Keywords:

Cutting  
Milling  
Chatter

## ABSTRACT

Critical comparison is presented related to the stability behaviour of milling processes performed by conventional, variable helix and serrated milling tools. The paper presents a general milling model linked to any non-proportionally damped dynamic system. Extended multi frequency solution and semi-discretization are implemented and used to calculate the stability of stationary milling. Measurements performed in industrial environment validate the general numerical algorithm that is able to predict the stability conditions of milling processes carried out by cylindrical cutters of optional geometry. Both the calculations and the measurements confirm that, for roughing operations, the highest stability gain can be achieved by serrated cutters. It is also demonstrated that variable helix milling tools can achieve better stability behaviour only if their geometry is optimized for the given cutting operation.

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## 1. Introduction

One of the main limitations of the utilization of high performance machining centres is the occurrence of self-excited vibrations during the cutting operations known in the literature as chatter. With the appearance of reliable and robust machining centres, which are capable to deliver high power and large forces to the cutting zone, the prediction of undesired vibrations becomes more and more important. The foundations of the theory of machine tool chatter were laid by Tlustý and Tobias [1,2] in the fifties. They showed that the main source of these vibrations is the so-called regenerative effect: the cutting edge interferes with its own past oscillation recorded on the wavy surface cut on the workpiece. For milling processes, the cutting edge cuts the surface that has been modulated by the previous cutting edge one tooth-past period earlier. Thus, the mathematical models for these phenomena are time-periodic delay-differential equations (DDE).

Milling tool designers and manufacturers have faced the task of reducing the regenerative effects by detuning the occurring time delays relative to each other in these dynamical systems. This led to the appearance of non-uniform pitch angle tools, variable helix tools and serrated tools. While these tools are expected to have improved properties from a vibrations viewpoint, the precise calculation and prediction of the stability properties of the corresponding milling processes is difficult and intricate, while also very sensitive to the precision of the dynamics modelling.

Several methods were developed to analyze the stability of milling processes. Frequency-domain methods, such as the single- and

multi-frequency solution [3,4,24] or the extended multi-frequency solution (EMFS) [5] and time-domain methods, such as the semi-discretization method (sdm) [6], the subspace iteration technique [7] or the solution in terms of Chebyshev polynomials [8] are valid examples. The advantage of frequency-domain methods is that they can directly use the frequency response function obtained by modal tests, while for time-domain methods the modal parameters must be determined by sophisticated procedures of modal analysis.

In order to derive the stability properties of the cutting operation, the tool geometry, the cutter-workpiece engagement and the cutting force characteristics should be described properly [9–12]. In the current paper, stability diagrams are determined for three different milling tools: a conventional, a variable helix and a serrated milling tool. It is shown that improvements in the stability properties can be achieved only if the tool geometry and the cutter-workpiece engagement are perfectly known and the machining parameters are tuned appropriately. The stability analysis is performed both by EMFS [5] and SDM [6], and the results are compared to cutting tests for all the three tools. The effect of the accuracy of the modal analysis that provides the input for SDM is also investigated. It is shown that for some particular cases, the predicted stability diagram is very sensitive to small changes in the modal analysis procedure.

## 2. General milling model for cylindrical tools

The model used in this study is based on [9], where theoretical studies were presented related to general cylindrical milling tools with number  $Z$  of flutes. As further extension, this paper provides an experimental confirmation of the results in [9]. The general geometrical model takes into account intricate geometrical features: (i) non-uniform constant helix angles  $\eta_i$  ( $i = 1, 2, \dots, Z$ )

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that affect the lag angles  $\varphi_{\eta_i}(z) = 2z/D \tan \eta_i$  along the axial  $z$  axis of the tool of diameter  $D$ ; (ii) continuously varying pitch angles  $\varphi_{p,i}(z) = \varphi_{p,i,0} + \varphi_{\eta_i}(z) - \varphi_{\eta_{i+1}}(z)$ ; (iii) variations in local radii as  $R_i(z) = D/2 - \Delta R_i(z)$ , which results in variations in local lead angles described by  $\kappa_i(z) = \cot^{-1}(dR_i(z)/dz)$ . This model leads to continuous delay functions  $\tau_{i,l}(z)$ , which can be expressed with the help of the continuous pitch angles as

$$\tau_{i,l}(z) = \frac{1}{\Omega} \sum_{k=1}^{l-1} \varphi_{p,(i+k) \bmod} z(z), \quad (1)$$

where  $\Omega$  is the constant angular velocity of the tool.

To explore the missed-cuts caused by radii variations, the effective geometrical chip thickness is evaluated as the minimum of all geometrically possible chip thicknesses in the form  $h_{g,i,e}(z,t) = \min_i h_{g,i}(z,t)$  [13], which can still be a negative value. The screen function  $g_i(z,t)$  takes into account the radial immersion and also the missed-cut effect, which is 1 for  $\varphi_{en} < (\varphi_i(z,t) \bmod 2\pi) < \varphi_{ex}$  and  $h_{g,i,e}(z,t) > 0$ , otherwise it is 0. The angles  $\varphi_{en}$  and  $\varphi_{ex}$  are the entry and exit angles measured clockwise from the  $y$  axis (see Fig. 2). The physical chip thickness at the  $i$ th flute at level  $z$  is then defined by the non-negative  $h_i(z,t) = g_i(z,t)h_{g,i,e}(z,t)$ .

The cutting force per unit axial depth of cut (also called specific cutting force) at a particular edge location is expressed in the  $tra$  coordinate system as  $\mathbf{f}_{tra,i}(z,t) = -\mathbf{f}(h_i(z,t))$ , where  $\mathbf{f}(h)$  is the empirical specific cutting force characteristics measured in all  $tra$  directions. Then, the resultant cutting force  $\mathbf{F}$  is the sum of the infinitesimal cutting forces along the elementary arc lengths  $ds_i = dz/\cos \eta_i/\sin \kappa_i(z)$  of the  $i$ th edge that are in cut. Thus

$$\mathbf{F}(t, \mathbf{r}_t(\theta)) = \sum_{i=1}^Z \int_0^{a_p} \gamma_i(z, t, \mathbf{r}_t(\theta)) dz, \quad (2)$$

where  $\gamma_i(z, t, \mathbf{r}_t(\theta)) = g_i(z, t) \mathbf{T}_i(z, t) \mathbf{f}_{tra,i}(z, t) / \cos \eta_i / \sin \kappa_i(z)$  with transformation matrix  $\mathbf{T}_i(z, t)$  given in [14]. Here,  $\mathbf{r}_t(\theta) = \mathbf{r}(t + \theta)$ , ( $\theta \in [-\sigma, 0]$ ,  $\sigma = \max_{i,l,z} \tau_{i,l}(z)$ ) expresses the typically intricate regeneration properties related to delays (1) (see details in [15]).

### 3. Process stability

Due to the possible intricate form of the regenerative delay and the general non-proportionally damped dynamics the system can be described with a first order delay differential equation (DDE)

$$\dot{\mathbf{q}}(t) - [\lambda_k] \mathbf{q}(t) = \mathbf{T}^T \mathbf{F}(t, \mathbf{T} \mathbf{q}_t(\theta)) \quad (3)$$

in the modal space described by  $\mathbf{q}$ , where the modal parameters determines  $\lambda_k = -\omega_{n,k} \xi_k + \omega_{d,k} i$  with undamped  $\omega_{n,k}$  and damped  $\omega_{d,k} = \omega_{n,k} (1 - \xi_k^2)^{1/2}$  natural angular frequencies and the damping ratio  $\xi_k$  of the  $k$ th mode. Eq. (3) is periodic by considering a specific state as  $\mathbf{F}(t, \bullet) = \mathbf{F}(t + T, \bullet)$ , where  $T$  is the principle period, which is integer multiple of the theoretical tooth passing period  $T_Z = 2\pi/\Omega/Z$ . Consequently, (3) has periodic solution  $\mathbf{q}_p(t) = \mathbf{q}_p(t + T)$ , which describes stable stationary cutting through the modal transformation matrix  $\mathbf{T}$ , i.e.,  $\mathbf{r}_p = \mathbf{T} \mathbf{q}_p$ . When stability is investigated the stability of the stable stationary cutting is of interest. Using perturbation around  $\mathbf{q}_p$  as  $\mathbf{q} = \mathbf{q}_p + \mathbf{u}$  a linearized so-called variational system [16] can be derived.

It is important to emphasize that the stability calculation is only performed on the local linearized dynamics described by  $\mathbf{u}$ . This means the frequency spectrum relates straight to the onset section of the emerging unstable motion, only. Consequently, during the experiment, when the vibration has been evolved more details can be recognized in the collected data. This is because the tool has reached such high threshold amplitude that it actually leaves the surface supposed to cut. This effect introduces different spectrum related to possibly complex quasi-periodic and chaotic motion [22], however this measured spectrum contains strong traces of spectra of both the forced stationary motion and the evolving unstable motion.

### 3.1. Stability prediction

There are two significant methods used in this work to determine the stability of stationary milling  $\mathbf{r}_p = \mathbf{T} \mathbf{q}_p$ . Both methods use the Floquet theorem [16] of linear time-periodic systems, but in quite different ways. Frequency domain methods [3,4] like EMFS [5] are based on considering a trial function involving time periodic coefficients and an exponential part related to the vibration frequency of the evolving vibration in the form  $\mathbf{u}(t) = \mathbf{A}(t)e^{i\omega t}$  where  $\mathbf{A}(t) = \mathbf{A}(t + T)$ . This introduces infinitely many harmonics separated by the multiples of the principle frequency  $\omega_T = 2\pi/T$ . Classic stability charts are 3-parameter problems in the parameter space  $(\Omega, a_p, \omega)$  and they are determined by the non-hyperbolic solutions when eigenvalues are crossing the imaginary axis of the complex plane. This means that the solutions related to the real stability boundaries need to be selected among many fake solutions that can be done by Cauchy's Argument Principle as in [5].

Time domain solutions like SDM are driven by the statement of Floquet theorem [16] that the solution state in the principle period is linearly dependent on the initial state. In other words [6],  $\mathbf{z}_{j+r} = \Phi \mathbf{z}_j$  where the state is represented by the hyper-vector  $\mathbf{z}_j = \text{col}(\mathbf{u}_j, \mathbf{u}_{j-1}, \dots, \mathbf{u}_{j-k})$  with  $\mathbf{u}_{j-k} = \mathbf{u}(t_j - k\Delta\theta)$  above the interval  $\theta \in [-\sigma, 0]$  in (2) with time steps  $\Delta\theta$  and integer  $k = \lceil \sigma/\Delta\theta \rceil$ . The principle period is discretized by  $T = r\Delta t$  with the same time-step  $\Delta t = \Delta\theta$ , and the integer resolution number is  $r$ . The transition matrix  $\Phi$  is determined by the step matrices  $\mathbf{B}_j$  [6], which maps the state to the next time step:  $\mathbf{z}_{j+1} = \mathbf{B}_j \mathbf{z}_j$ . In contrast to the frequency domain solutions, the asymptotic stability of the stationary cutting can be determined by checking the eigenvalues (characteristic multipliers)  $\mu$  of the transition matrix  $\Phi$ . The stationary solution of the original system (3) undergoes a secondary Hopf bifurcation if a complex pair of critical multipliers  $\mu_c$  crosses the unit circle. Period doubling (flip) or cyclic fold (saddle-node) bifurcation occurs for  $\mu_c = -1$  or  $\mu_c = 1$ , respectively. The resulting stability charts are two dimensional problems in the parameter space  $(\Omega, a_p)$ .

The stability charts are produced by both methods in order to check each other. The actual identification of the intricate stability boundaries is carried out by the multi-dimensional bisection method [18] that can process the resulting data sets of both cases. Along the stability limits, the frequencies and harmonics of the evolving vibrations are also determined.

### 3.2. Experimental chatter identification

To construct some preliminary stability charts, the orthogonal to oblique model for Al5086 material was adapted from [14]. Based on these preliminary stability charts, it was possible to select the cutting parameters. The real test conditions required the avoidance of sudden or unexpected extreme vibrations, so the values  $a_p$  of the axial depth of cut were increased step-by-step.

The nature of the vibrations was primarily decided by using the signals of accelerometers attached to the spindle housing. These signals were transformed to linearly scaled velocity time/frequency (waterfall) diagrams directly in a Pulse B&K data acquisition system [22]. The stable stationary cutting was recognized by its special regular harmonic pattern separated by the principle frequency  $\omega_T$ . Note that small (cca. < 10%) participation of the chatter frequency  $\omega$  (and their harmonics) were still allowed close to the presumable stability limits due to the parasite excitation induced by some noise. In these cases, the machined workpiece surface pattern contained regular bites related to the feed motion only (see Fig. 1a), and these showed a good correlation with the above analysis of the vibration signal.

Chattering motions were easily identified via the strong appearance of the chatter frequency  $\omega$  (and their harmonics) that grew far above the harmonics of the forced vibration. The machined workpiece surface patterns were inspected visually and the irregularities corresponding to chatter were identified (Fig. 1b). In many cases, however, the vibrations were so large and

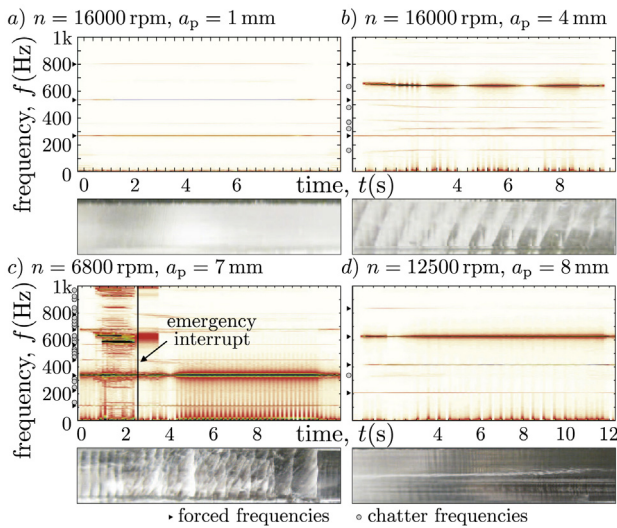


Fig. 1. Vibration pattern and corresponding surface marks in case of stable stationary cutting (a), chatter vibration (b), extreme chaotic chase (c) and large amplitudes of stable cutting (d).

almost immediately chaotic, that the stationary cutting could be reported as unstable without any further need for visual inspection (see Fig. 1c).

When the integer multiple of the principle excitation frequency  $\omega_T$  is close to a natural frequency, that is, resonance occurs, the chatter frequency  $\omega$  and its harmonics can be hidden by or close to the ‘resonant’ forced frequencies. While quite large vibration amplitudes can be experienced, still stable stationary vibration is reported if the inspected machined surface presents a regular pattern (see Fig. 1d) as reported also in [23].

#### 4. Measurements circumstances

The regenerative model and the applied algorithms were validated by experiments. A three-axis milling machine was used with three different cylindrical carbide milling tools of the same diameter (see Fig. 2). The cutting performances were compared for a conventional tool, a non-uniform constant helix tool, and a serrated one.

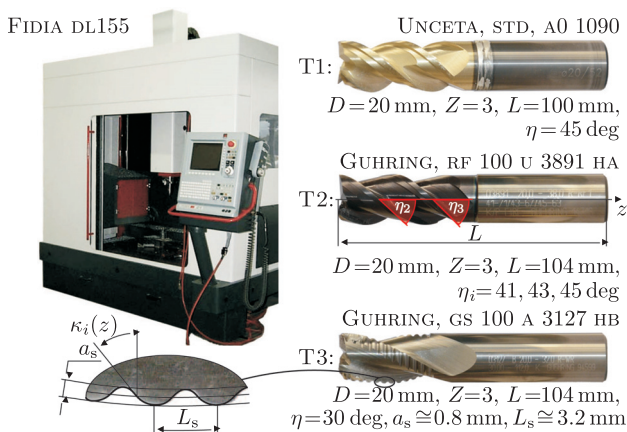


Fig. 2. Three-axis FIDIA milling machine and the cylindrical milling tools used for the chatter tests.

The FRF matrix  $\mathbf{H}(\omega)$  at the tool tip was measured in a special automatic way (see [20]), and confirmed manually. Then, modal parameters were extracted from pole-stability diagrams built by means of the Rational Fraction Polynomial Method  $R_{FFM}$  [19]. This algorithm uses high DOF mechanical models that result in the poles  $\lambda_k$ , the modal scaling factors  $Q_k$  and the geometrically normalized modeshapes  $\mathbf{p}_k$ .

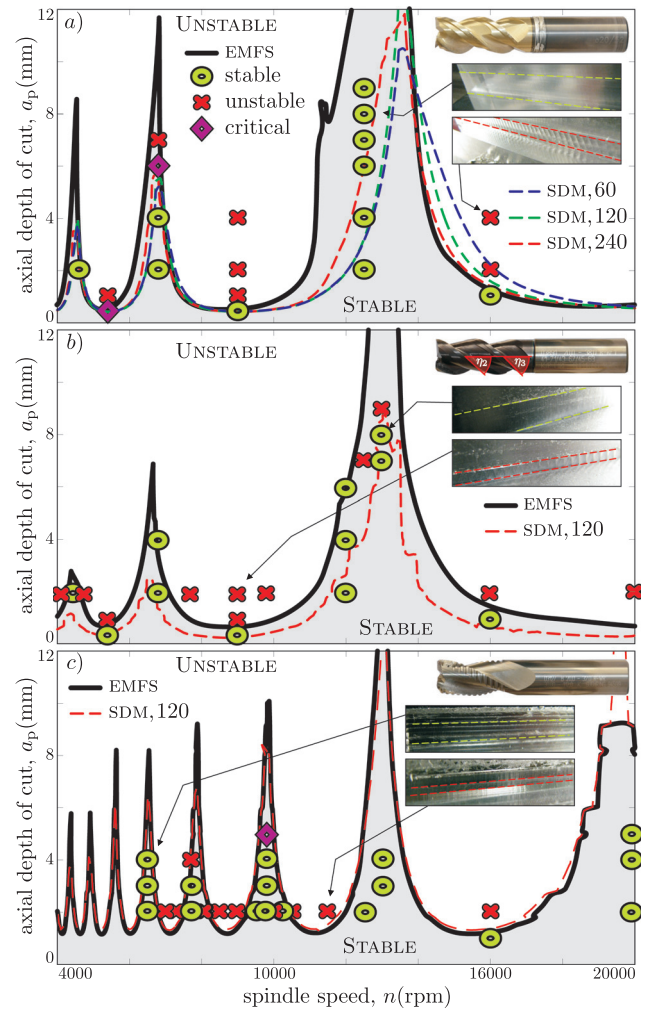


Fig. 3. Stability charts predicted for conventional helical (a), non-uniform constant helix (b), and serrated (c) milling tools by means of EMFS and SDM (colored dashed lines referring to different DOF's applied in RFFM), and the results of chatter tests.

Since the quality of the stability predictions by SDM strongly depends on the quality of the FRF fitting, discrepancies were expected between the predictions by EMFS and by SDM. Note that EMFS uses the measured FRF directly, i.e. without any need for fitting. Good agreement of EMFS and SDM with fitted FRF was obtained only for the serrated cutter (see Fig. 3c), while for the other two cutters the stability boundaries were somewhat different. The differences in accuracy were traced w.r.t. the DOF used in RFFM for the conventional milling tool. In Fig. 3a, the results are indicated by coloured dashed lines. Regarding the quality of FRF fitting, the overall relative errors in direct measurements: 14.4% for 60DOF, 11.58% for 120DOF and 9.51% for 240DOF models, while only 20–30 relevant DOF's were used for the stability calculations in all cases.

Regardless of the sensitivity of the SDM on FRF fitting or the sensitivity of the EMFS w.r.t. vibration measurement data, the efficient spindle speed zones were predicted accurately by both methods and these were also confirmed by cutting tests (as it is shown in Fig. 3). Although the level of the stationary vibration  $\mathbf{r}_p$  increases in these efficient pockets, the surface quality is still granted, that is, finishing operations might be bypassed.

As shown in Fig. 3b, the non-uniform constant helix tool may increase the minimum of the stability boundaries (instability lobes) by cca. 50%, but the stable pockets of high efficiency cutting become 3 times narrower, so the overall stability properties are practically not better than those of its conventional counterpart; the measurement tests confirmed this observation.

The serrated tool was used in the last series of measurements. This was the most successful test since both EMFS and SDM predicted the same stability limits (see Fig. 3c). The measurements

confirmed that the application of serrated cutters leads to the appearance of many new (still narrow) stable pockets as predicted in [14]. Also, the minimum of the instability lobes is increased by about a factor of 3 from 0.5 mm up to 1.5 mm.

## 5. Summary

Since the importance of reducing the regenerative effect in cutting processes is recognized, milling tool developers and manufacturers have designed and marketed many non-conventional cylindrical milling tools based on the idea of messing up the occurring time delays in the milling process. Generally speaking, these tools improve chatter avoidance, but due to their intricate geometric structure, reliable prediction of the increased chatter-free parameters is extremely difficult. This task is especially challenging since the stability boundaries are sensitive for the precise modal testing of the machine tool.

To overcome these difficulties, two principally different methods were implemented: semi-discretisation and multi-frequency solution. Both provided reasonably identical stability charts.

The comparison of 3 different helical milling tools of identical cylindrical geometry showed that the serrated tool provided the most significant improvement in cutting stability: both the minima of the lobes and the number of stable pockets were increased, while the non-uniform constant helix angle tool presented only slight increase of lobe minima and narrowed the stable pockets compared to a conventional helical tool. On the other hand, serrated tools are to be used for roughing operations only, while the optimized variable helix tools may provide significantly better stability.

## Acknowledgements

This research was partially supported by the Hungarian National Science Foundation under grant no. OTKA K105433, K101714 and K108779, by National Excellence Programme (TÁMOP 4.2.4. A/-11-1-2012-0001), by the EU FP7 DYNXPERS (260073/FP7-2010-NMP-ICT-FoF) and the European Research Council 2013 Advanced Grant under project no. ERC-2013-AdG/340889 (SIREN).

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